Corrigendum to:
Operator monotone functions and Łöwner functions of several variables

By Jim Agler, John E. McCarthy, and N. J. Young

Abstract

We fix a gap in the proof of Theorem 7.24 in Ann. of Math. 176 (2012), 1783–1826.

There is a gap in the proof of Theorem 7.24 in [1], though the statement of the theorem is correct.

In the proof of necessity, we argue that Λ is in $G$ by contradiction. If it were not, invoking the Hahn-Banach separation theorem would yield a real skew-symmetric matrix $K$ and a constant $δ ≥ 0$ such that $\text{tr}(ΓK) ≥ −δ$ for all $Γ$ in $G$, and $\text{tr}(ΛK) < −δ$. In the proof we assumed that $δ = 0$, but this assumption is unjustified.

Instead, we argue as follows. Define $∆$ by

$$∆^r_{ij} = (x^r_j−x^r_i)K_{ji}, \quad i \neq j,$$

and with the diagonal entries $∆^r_{ii}$ chosen so that each $∆^r ≥ 0$ and so that

$$µ^r := \sum_{i=1}^n f_{r,i}∆^r_{ii}$$

is minimal over all choices of $∆^r_{11}, \ldots, ∆^r_{nn}$ such that $∆ ≥ 0$. (A minimal choice exists, since all the $f_{r,i}$ are strictly positive by assumption.) Then $∆$ is in $SAM^d_n$, and

$$[∆^s, S^r]_{ij} = (x^s_j−x^s_i)K_{ji}(x^r_j−x^r_i) = [∆^r, S^s]_{ij}.$$
As $f$ is locally $M_n$ monotone, we must have then that $D\Delta f(S) \geq 0$ by Lemma 7.3. As

$$-\delta > \text{tr}(AK) = \sum_{1 \leq i,j \leq n} [D\Delta f(S)]_{ij} - \sum_{r=1}^d \sum_{i=1}^n \Delta_{ri}^r f_{r,i},$$

we get that

$$(0.2) \sum_{r=1}^d \mu^r - \delta > \sum_{1 \leq i,j \leq n} [D\Delta f(S)]_{ij} \geq 0.$$ 

By Duffin’s strong duality theorem [2], the minimum $\mu^r$ in (0.1) satisfies

$$(0.3) -\mu^r = \min \sum_{i \neq j} \Delta_{ij} A^r(i,j),$$

where $A^r$ range over the set of real positive matrices such that the diagonal entries of $A^r$ are $f_{r1}, \ldots, f_{rn}$ for each $r$.

For each such $A = (A^1, \ldots, A^d)$, let $\Gamma$ be the corresponding element of $\mathcal{G}$: $\Gamma_{ii} = 0$ and

$$\Gamma_{ij} = \sum_{r=1}^d (x_{ji}^r - x_{ij}^r) A^r(i,j) \quad \text{for } i \neq j.$$ 

We have

$$-\delta \leq \text{tr} \Gamma K$$

$$= \sum_{i \neq j} \sum_{r=1}^d (x_{ji}^r - x_{ij}^r) A^r(i,j) K_{ji}$$

$$= \sum_{r=1}^d \sum_{i \neq j} \Delta_{ij}^r A^r(i,j).$$

Hence, by equation (0.3), $-\delta \leq \sum_{r=1}^d (-\mu^r)$, so $\sum_{r=1}^d \mu^r \leq \delta$. This contradicts (0.2), so it follows that $\Lambda \in \mathcal{G}$, and necessity is proved.

References


CORRIGENDUM

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University of California San Diego, La Jolla, CA
http://math.ucsd.edu/people/faculty/Jim-Agler/

Washington University, St. Louis, MO
E-mail: mccarthy@math.wustl.edu
http://www.math.wustl.edu/~mccarthy/

Leeds University, Leeds, United Kingdom and
Newcastle University, Newcastle upon Tyne, United Kingdom
E-mail: N.J.Young@leeds.ac.uk
http://www1.maths.leeds.ac.uk/~nicholas/