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Corrigendum to "A smooth foliation of the 5-sphere by complex surfaces"

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Our paper [MV02] claims that it describes a foliation of $\mathbb{S}^5$ by complex surfaces. However it was pointed out to us by the anonymous referee of a related article that the foliation constructed in the paper lives in fact on a 5-manifold with nontrivial fundamental group.

We observe that, even with this modification, this foliation is still the first example of such an exotic CR-structure.

We use the notations and results of [MV02] and assume that the reader is acquainted with them. We refer to [MV11] for full details of what follows.

The foliation of [MV02] is obtained by gluing, thanks to Lemma 1, two tame foliations on manifolds with boundary. The second one, $\mathcal{N}$, is supposed to be diffeomorphic to $K \times \overline{\mathbb{D}}$. However, the foliated $\mathcal{N}$ which is constructed by quotient is in fact diffeomorphic to the $\overline{\mathbb{D}}$-bundle associated to L. They are definitely not diffeomorphic, since the first manifold has a nilpotent fundamental group (see [MV02, §1.2]), whereas the second one retracts on $\mathbb{S}^1 \times \mathbb{S}^1 \times \mathbb{S}^1$.

When gluing $\mathcal{M}$ and this "new" $\mathcal{N}$, one does not obtain the 5-sphere. Let

$$F^p = \{[z_0 : z_1 : z_2 : z_3] \in \mathbb{P}^3 \quad | \quad z_1^3 + z_2^3 + z_3^3 = z_0^3\}.$$

Then, what is really proved in [MV02] is the following theorem.

Theorem. *Let Z be the 5-dimensional bundle over the circle with fiber F^p and monodromy the multiplication by $\omega = \exp 2i\pi/3$ on the affine part.*

There exists on Z an exotic smooth, codimension-one, integrable and Levi-flat CR*-structure. The induced foliation by complex surfaces satisfies*:

(i) *There are only two compact leaves both biholomorphic to an elliptic bundle over $\mathbb{E}_\omega$. Since this surface has odd first Betti number it is not Kähler.*

(ii) *One compact leaf is the boundary of a compact set in Z whose interior is foliated by line bundles L over $\mathbb{E}_\omega$ with Chern class -3. The two compact leaves are the boundary components of a collar and the leaves in the interior of this collar are biholomorphic to W, the principal $\mathbb{C}^*$-bundle associated to L.*

(iii) *The other leaves have the homotopy type of a bouquet of eight copies of* $\mathbb{S}^2$ *and they are all biholomorphic to the affine complex smooth manifold* $F^p \cap \mathbb{C}^3$.

References

[MV02] L. Meersseman and A. Verjovsky, A smooth foliation of the 5-sphere by complex surfaces, *Ann. of Math.* **156** (2002), 915–930. MR 1954239. Zbl 1029.32019. http://dx.doi.org/10.2307/3597286.

[MV11] ———, Correction to "a smooth foliation of the 5-sphere by complex surfaces", 2011. arXiv 1106.0504.

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