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Erratum: Rotation invariant valuations on convex sets*

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1) In the above cited article [1] the proof of Theorem A contains a gap. We do not know if Theorem A is true (see however discussion below on a proof of this result under an additional assumption). Nevertheless this gap does not influence any other result proved in the article. Other results remain correct, and their proofs are independent of the proof of Theorem A (in particular Theorem B and results of Sections 5,6). Also this gap does not influence the results of the subsequent article [2] which was based on Theorem B of [1] only. A revised version of Theorem A is stated below.

2) Part of the remark on p. 1001 is not correct. There it is written: "However, for higher derivatives in ε $\frac{d^j}{d\varepsilon^j}\big|_{\varepsilon=0}\int_{K+\varepsilon B}|s|^{2q}ds$ (at least for even j) the similar monotonicity property on the class of convex compact sets containing 0 fails to be true (even in the 1-dimensional case)." In fact in the 1-dimensional case this monotonicity is satisfied for trivial reasons. In higher dimensions it is still unknown. This remark was just a side remark; it was used nowhere in the article and in no subsequent work. Note also that the previous sentence in this remark is correct. It says: "The valuation $\phi(K) := \frac{d}{d\varepsilon}\big|_{\varepsilon=0}\int_{K+\varepsilon B}|s|^{2q}ds = \int_{\partial K}|s|^{2q}d\sigma_K(s)$ is monotone in the following sense: if $K_1 \supset K_2 \ni 0$ then $\phi(K_1) \geq \phi(K_2) \geq 0$."

Let us state a revised version of Theorem A following [3]. Let $\mathbb{R}^d$ be the Euclidean space. Let $\mathcal{K}^d$ denote the family of convex compact subsets of $\mathbb{R}^d$. Recall that equipped with the Hausdorff metric, $\mathcal{K}^d$ is a locally compact space. Theorem A in [1] says that every continuous $\mathrm{SO}(d)$- (resp. $\mathrm{O}(d)$-) invariant valuation can be approximated uniformly on compact subsets of $\mathcal{K}^d$ by *polynomial* continuous $\mathrm{SO}(d)-$ (resp. $\mathrm{O}(d)-$) invariant valuations. Recently in [3], Corollary 3.1.9, we were able to prove Theorem A under an additional assumption of *quasi-smoothness* of valuations. Let us describe this notion.

Definition 1. Let $\phi\colon \mathcal{K}^d \to \mathbb{C}$ be a continuous valuation. It is called *quasi-smooth* if the map

$$K \mapsto [(t,x) \mapsto \phi(tK + x)],$$

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$K \in \mathcal{K}^d, x \in \mathbb{R}^d, t \in [0,1]$, is a continuous map $\mathcal{K}^d \to C^d([0,1] \times \mathbb{R}^d)$ (where C^d denotes the space of d times continuously differentiable functions).

In [3] we defined a natural Fréchet topology on the space $\mathrm{QV}(\mathbb{R}^d)$ of quasi-smooth valuations. It was shown that continuous polynomial valuations are quasi-smooth.

Theorem 2 ([3, Cor. 3.1.8]). *The subspace of continuous polynomial valuations is dense in the space of quasi-smooth valuations.*

Corollary 3 ([3, Cor. 3.1.9]). *Let G be a compact subgroup of $\mathrm{GL}_d(\mathbb{R})$. Then G-invariant polynomial valuations are dense in the space of G-invariant quasi-smooth valuations.*

In particular, taking $G = \mathrm{SO}(d)$ or $G = \mathrm{O}(d)$ in Corollary 3, we get Theorem A under the assumption of quasi-smoothness.

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References

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[3] ———, Theory of valuations on manifolds, I. Linear spaces, *Israel J. Math.* **156** (2006), 311–399.

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