Annals of Mathematics, **165** (2007), 665–673

Corrigendum: Self-dual instantons and holomorphic curves

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Abstract

We correct two mistakes in [1]. The first concerns the exponential decay in the proof of Theorem 7.4 and the second concerns the bubbling argument in the proof of Theorem 9.1.

1. Exponential decay

For Theorem 7.1. Replace the hypothesis $\|B_t\|_{L^\infty(\Omega\times\Sigma)} + \varepsilon\,\|C\|_{L^\infty(\Omega\times\Sigma)} \le c_0$ on p. 615 by the weaker assumption

$$\sup_{(s,t)\in\Omega} \|B_t(s,t)\|_{L^2(\Sigma)} + \varepsilon \sup_{(s,t)\in\Omega} \|C(s,t)\|_{L^2(\Sigma)} \le c_0. \tag{1}$$

All the estimates in the proof of Theorem 7.1 continue to hold under this assumption. To see this, use the inclusion $W^{1,2}(\Sigma) \hookrightarrow L^4(\Sigma)$ to obtain inequalities of the form

$$\|B_t\|_{L^4(\Sigma)}\,\|C\|_{L^4(\Sigma)} \le c\sqrt{u_0 v_0}, \qquad \|B_t\|^2_{L^4(\Sigma)} \le v_0 + cu_0,$$

where u_0, v_0 are as in the proof of Theorem 7.1.

Corollary 1.1. *Let $\Omega \subset \mathbb{C}$ be an open set and $K \subset \Omega$ be a compact subset. Then for every constant $c_0 > 0$, there exist constants $\varepsilon_0 > 0$ and $c > 0$ such that the following holds. If $0 < \varepsilon \le \varepsilon_0$ and $\Xi = A + \Phi\,ds + \Psi\,dt$ is a connection on $\Omega \times \Sigma$ that satisfies*

$$\begin{aligned} \partial_t A - \mathrm{d}_A\Psi + *_s(\partial_s A - \mathrm{d}_A\Phi - X_s(A)) &= 0, \\ \partial_t\Phi - \partial_s\Psi - [\Phi,\Psi] + \varepsilon^{-2} * F_A &= 0, \end{aligned} \tag{2}$$

and (1) *holds, then*

$$\|B_t\|_{L^\infty(K\times\Sigma)} + \varepsilon\,\|C\|_{L^\infty(K\times\Sigma)} \le c\left(\|B_t\|_{L^2(\Omega\times\Sigma)} + \varepsilon\,\|C\|_{L^2(\Omega\times\Sigma)}\right).$$

Proof. By Theorem 7.1 (in the above strengthened form), the connection Ξ satisfies (7.4) in [1, p. 615]. The assertion follows by taking $p = \infty$ and using [1, Lemma 7.6] with $p = 4$. □

For Lemma 7.5. On page 620 replace the inequality (7.7) by

$$\begin{aligned}&\|\alpha\|^2+\|\phi\|^2+\|\psi\|^2\\ &\quad\le c\Big(\|{*_s}\nabla_s\alpha-{*_s}\mathrm{d}X_s(A)\alpha-{*_s}\mathrm{d}_A\phi-\mathrm{d}_A\psi\|^2\\ &\qquad+\varepsilon^2\left\|\nabla_s\psi-\varepsilon^{-2}\mathrm{d}_A\alpha\right\|^2+\varepsilon^2\left\|\nabla_s *_s\phi+\varepsilon^{-2}\mathrm{d}_A *_s\alpha\right\|^2\Big).\end{aligned}$$

On page 621 replace the last two sentences in the proof of Lemma 7.5 by the following text.

Hence it follows from Lemma 7.3 and Lemma 7.4 in [10] that there exist constants $\varepsilon_0>0$, $\nu_0\in\mathbb{N}$, and $c>0$ such that the estimate (7.7) holds with $0<\varepsilon\le\varepsilon_0$ and $A+\Phi\,\mathrm{d}s$ replaced by $A_\nu+\Phi_\nu\,\mathrm{d}s$ where $\nu\ge\nu_0$ (here the estimate for α follows from Lemma 7.4 and the estimates for ϕ and ψ from Lemma 7.3). With $\varepsilon=\varepsilon_\nu$ and $\nu>c$ this contradicts our assumption. □

Proof of Theorem 7.4. The last displayed inequality on p. 622 is correct as it stands; however its proof uses Corollary 1.1 above.

Replace the first displayed inequality on p. 623 by

$$\|B_t\|^2+\|C\|^2\le c_3\left(\|\nabla_s B_t-\mathrm{d}X_s(A)B_t-\mathrm{d}_A C\|^2+\varepsilon^{-2}\|\mathrm{d}_A B_t\|\right).$$

(The mistake in [1] is the factor ε^2 in front of $\|C\|^2$ in this inequality; it can be removed because of the improved inequality in Lemma 7.5.) Inspection of the formula for $f''(t)$ shows that this stronger estimate is needed to prove the inequality $f''(t)\ge\rho^2 f(t)$ for $t\ge 1$ (use the expression after the fourth equal sign in the formula for $f''(t)$ on page 622). □

2. An *a priori* estimate

The following *a priori* estimate is an adaptation of [2, Lemma 9.1] to the present context. It is needed in the proof of Theorem 9.1.

LEMMA 2.1. *There is a constant $\delta_0>0$ with the following significance. Let $\Omega\subset\mathbb{R}^2$ be an open set and $K\subset\Omega$ be a compact subset. Then, for every $c_0>0$ and every $p\ge 2$, there are positive constants ε_0 and c such that the following holds. If $0<\varepsilon\le\varepsilon_0$ and the maps $A:\Omega\to\mathcal{A}(P)$ and $\Phi,\Psi:\Omega\to\Omega^0(\Sigma,\mathfrak{g}_P)$ satisfy* (2) *and*

$$\|\partial_t A-\mathrm{d}_A\Psi\|_{L^\infty(\Omega\times\Sigma)}\le c_0,\qquad \|F_A\|_{L^\infty(\Omega\times\Sigma)}\le\delta_0, \tag{3}$$

then

$$\int_K\left(\|F_A\|^p_{L^2(\Sigma)}+\varepsilon^p\|\nabla_s F_A\|^p_{L^2(\Sigma)}+\varepsilon^p\|\nabla_t F_A\|^p_{L^2(\Sigma)}\right)\le c\varepsilon^{2p}, \tag{4}$$

$$\sup_K\left(\|F_A\|_{L^2(\Sigma)}+\varepsilon\|\nabla_s F_A\|_{L^2(\Sigma)}+\varepsilon\|\nabla_t F_A\|_{L^2(\Sigma)}\right)\le c\varepsilon^{2-2/p}. \tag{5}$$

Proof. As in [1, Lemma 7.6] one can show that there exist constants $\delta_0 > 0$ and $c_1 > 0$ such that every $A \in \mathcal{A}(P)$ with $\|F_A\|_{L^\infty(\Sigma)} \le \delta_0$ satisfies the inequalities

$$\|\phi\| \le c_1 \|\mathrm{d}_A\phi\|,$$

$$\|\mathrm{d}_A(*_s\mathrm{d}X_s(A)\alpha + \dot{*}_s\alpha)\| \le c_1(\|\alpha\| + \|\mathrm{d}_A\alpha\| + \|\mathrm{d}_A *_s \alpha\|)$$

for $s \in \mathbb{R}$, $\phi \in \Omega^0(\Sigma; \mathfrak{g}_P)$, and $\alpha \in \Omega^1(\Sigma; \mathfrak{g}_P)$. Here and in the following all norms are L^2-norms on Σ.

Now let A, Φ, Ψ satisy the hypotheses of the lemma and define

$$B_s := \partial_s A - \mathrm{d}_A\Phi, \quad B_t := \partial_t A - \mathrm{d}_A\Psi, \quad C := \partial_t\Phi - \partial_s\Psi - [\Phi, \Psi]. \tag{6}$$

Then the proof of [1, Th. 7.1] shows that

$$\begin{aligned}\varepsilon^2(\nabla_s\nabla_s C + \nabla_t\nabla_t C) = {}& \mathrm{d}_A^{*_s}\mathrm{d}_A C - 2*[B_t \wedge B_t] + *[*_s X_s(A) \wedge B_t] \\ & - *\,\mathrm{d}_A(*_s\mathrm{d}X_s(A)B_t + \dot{*}_s B_t).\end{aligned}$$

Hence, with $\Delta := \partial^2/\partial s^2 + \partial^2/\partial t^2$ the standard Laplacian, we have

$$\begin{aligned}\Delta\|C\|^2 &= 2\|\nabla_s C\|^2 + 2\|\nabla_t C\|^2 + 2\langle\nabla_s\nabla_s C + \nabla_t\nabla_t C, C\rangle \\ &= 2\varepsilon^{-4}\|\mathrm{d}_A *_s B_t\|^2 + 2\varepsilon^{-4}\|\mathrm{d}_A B_t\|^2 + 2\varepsilon^{-2}\|\mathrm{d}_A C\|^2 \\ &\quad - 4\varepsilon^{-2}\langle C, *[B_t \wedge B_t]\rangle + 2\varepsilon^{-2}\langle C, *[*_s X_s(A) \wedge B_t]\rangle \\ &\quad - 2\varepsilon^{-2}\langle C, *\mathrm{d}_A(*_s\mathrm{d}X_s(A)B_t + \dot{*}_s B_t)\rangle \\ &\ge \frac{\delta}{\varepsilon^2}\|C\|^2 - \frac{c}{\varepsilon^2}\|C\|.\end{aligned}$$

The last inequality holds for $\varepsilon \le \varepsilon_0$, with ε_0 sufficiently small, and with suitable positive constants δ and c, depending only on δ_0, c_0, and c_1 (as well as the metrics on Σ and the vector fields X_s). Since $2\Delta\|C\|^p \ge p\|C\|^{p-2}\Delta\|C\|^2$ for $p \ge 2$, this implies

$$\|C\|^p \le \frac{c}{\delta}\|C\|^{p-1} + \frac{2\varepsilon^2}{p\delta}\Delta\|C\|^p.$$

Using the inequality $ab \le a^p/p + b^q/q$ with $1/p + 1/q = 1$, $a := c/\delta$ and $b := \|C\|^{p-1}$ we obtain $b^q = \|C\|^p$, and hence

$$\|C\|^p \le \frac{c^p}{\delta^p} + \frac{2\varepsilon^2}{\delta}\Delta\|C\|^p. \tag{7}$$

By [2, Lemma 9.2], this implies that

$$\int_{B_R(z)}\|C\|^p \le \frac{\pi(R+r)^2c^p}{\delta^p} + \frac{8\varepsilon^2}{r^2\delta}\int_{B_{R+r}(z)}\|C\|^p$$

for every $z \in \mathbb{C}$ and every pair of positive real numbers R and r such that $B_{R+r}(z) \subset \Omega$. Now observe that $\varepsilon^2\|C\| = \|F_A\| \le \delta_0\mathrm{Vol}(\Sigma)$ and use the last inequality repeatedly, with R replaced by $R + r, R + 2r, \ldots, R + (p-1)r$, to obtain the estimate $\int_{B_R(z)}\|C\|^p \le c_p$ for every $z \in \mathbb{C}$ such that $B_{R+pr}(z) \subset \Omega$.

Now choose R and r such that $B_{R+pr}(z) \subset \Omega$ for every $z \in K$. Cover K by finitely many balls of radius R to obtain

$$\int_K \|F_A\|^p = \varepsilon^{2p} \int_K \|C\|^p \le c_{K,p}\varepsilon^{2p}. \tag{8}$$

It follows from (7) that the function $z \mapsto \|C(z)\|^p + c^p |z - z_0|^2 / 8\delta^{p-1}\varepsilon^2$ is subharmonic in Ω for every $z_0 \in \mathbb{C}$. Hence, by the mean value inequality and (8), we have

$$\sup_K \|F_A\| = \varepsilon^2 \sup_K \|C\| \le c_{K,p}\varepsilon^{2-2/p} \tag{9}$$

for a suitable constant $c_{K,p}$. It follows from (8) and (9) that every connection $\Xi = A + \Phi\, ds + \Psi\, dt$ on $\Omega \times P$ that satisfies (2) and (3) also satisfies (1) in every compact subset of Ω and hence, by Corollary 1.1, satisfies the hypotheses of [1, Th. 7.1]. Hence it follows from [1, Th. 7.1] with $p = \infty$ that, for every open set U with $\mathrm{cl}(U) \subset \Omega$, there is a constant c_U such that every conection Ξ on $\Omega \times P$ that satisfies (2) and (3) also satisfies the estimates

$$\begin{aligned} \varepsilon \|\nabla_s B_t\|_{L^\infty(U\times\Sigma)} + \varepsilon \|\nabla_t B_t\|_{L^\infty(U\times\Sigma)} &\le c_U, \\ \varepsilon \|C\|_{L^\infty(U\times\Sigma)} + \varepsilon^2 \|\nabla_s C\|_{L^\infty(U\times\Sigma)} + \varepsilon^2 \|\nabla_t C\|_{L^\infty(U\times\Sigma)} &\le c_U, \\ \|C\|_{L^2(U\times\Sigma)} + \varepsilon \|\nabla_s C\|_{L^2(U\times\Sigma)} + \varepsilon \|\nabla_t C\|_{L^2(U\times\Sigma)} &\le c_U. \end{aligned} \tag{10}$$

Note that the last inequality is equivalent to (4) for $p = 2$.

Now consider the function $u : U \to \mathbb{R}$ defined by

$$u(s,t)^2 := \frac{1}{2}\left(\|C(s,t)\|^2 + \varepsilon^2 \|\nabla_s C(s,t)\|^2 + \varepsilon^2 \|\nabla_t C(s,t)\|^2 \right).$$

Again all norms are L^2-norms on Σ. In the following we shall assume, for simplicity, that the Hodge $*$-operator $*_s = *$ is independent of s and that $X_s = 0$ for all s. Then, as in the proof of [1, Th. 7.1],

$$\begin{aligned} \Delta u^2 = {} & \varepsilon^{-2} \|d_A C\|^2 + \|\nabla_s C\|^2 + \|\nabla_t C\|^2 + \|d_A \nabla_s C\|^2 + \|d_A \nabla_t C\|^2 \\ & + \varepsilon^2 \|\nabla_s \nabla_s C\|^2 + \varepsilon^2 \|\nabla_t \nabla_t C\|^2 + 2\varepsilon^2 \|\nabla_s \nabla_t C\|^2 \\ & - 2\varepsilon^2 \langle C, [\nabla_s C, \nabla_t C]\rangle - 2\varepsilon^{-2} \langle C, *[B_t \wedge B_t]\rangle \\ & - 4\langle \nabla_s C, *[B_t \wedge \nabla_s B_t]\rangle - 4\langle \nabla_t C, *[B_t \wedge \nabla_t B_t]\rangle \\ & + \langle d_A \nabla_s C, [B_s, C]\rangle + \langle d_A \nabla_t C, [B_t, C]\rangle \\ & - \langle \nabla_s C, *[B_s \wedge * d_A C]\rangle - \langle \nabla_t C, *[B_t \wedge * d_A C]\rangle. \end{aligned}$$

For ε sufficiently small it follows that

$$\Delta u^2 \ge \frac{\delta}{\varepsilon^2} u^2 - \frac{c}{\varepsilon^2} u,$$

with suitable positive constants δ and c. To see this examine the last eight terms in the formula for Δu^2 and use (10). Now it follows as in (7) that

$$u^p \le \frac{c}{\delta} u^{p-1} + \frac{2\varepsilon^2}{p\delta} \Delta u^p$$

for $p \geq 2$. By (9) and (10), we have $u \leq c'/\varepsilon$ for some constant c'. Hence we can argue as above to show that, for every compact subset $K \subset U$, there is a constant $c_{K,p} > 0$ such that $\int_K u^p \leq c_{K,p}$ and $\sup_K u^p \leq c_{K,p}\varepsilon^{-2}$. This proves the lemma. □

3. Bubbling analysis

The assertion on p. 634 that the limit connection Ξ_0 represents a *nonconstant* holomorphic sphere $S^2 \to \mathcal{M}(P)$ does not seem to follow from the argument in [1]. A modified bubbling argument does result in a nonconstant holomorphic sphere but only proves a weaker estimate; i.e., we must weaken the assertion of Theorem 9.1 and the assumption of Theorem 8.1. Then Theorem 9.2 remains valid.

For Theorem 8.1. The assertion of Theorem 8.1 in [1, p. 623] continues to hold if the hypothesis (8.1) is replaced by the weaker inequality

$$\varepsilon^{-1} \|F_A\|_{L^\infty} + \|\partial_t A - \mathrm{d}_A \Psi\|_{L^\infty} \leq c_0. \tag{11}$$

To see this, replace the last inequality on p. 625 by $\|C^\nu\|_{L^p} \leq c\varepsilon_\nu^{2/p-1}$ or, equivalently,

$$\|F_{A_\nu}\|_{L^p} \leq c\varepsilon_\nu^{1+2/p}.$$

For $p = 2$ this follows from the first inequality in Step 2 on page 625, for $p = \infty$ it holds by assumption, and for $2 \leq p \leq \infty$ it follows by interpolation. Now replace the constant ε_ν^2 by $\varepsilon_\nu^{1+2/p}$ in the following places.

- In the inequality (8.4) on page 626.
- Replace the inequality $\|A' - A\|_{L^p} \leq c_2\varepsilon^2$ by $\|A' - A\|_{L^p} \leq c_2\varepsilon^{1+2/p}$ in the middle of page 626.
- In the first two inequalities after (8.9), in the first inequality after (8.10), and in the first inequality in the proof of Step 5 (page 628).
- In the first inequality on page 629 and in the last inequality before (8.11).

The next lemma is a local version of Theorem 8.1; it is needed in the proof of Theorem 9.1. Let $\Omega_\nu \subset \mathbb{C}$ be an exhausting sequence of open sets and s_ν, $\varepsilon_\nu > 0$, $\delta_\nu > 0$ be seqences of real numbers such that $s_\nu \to s_0$, $\varepsilon_\nu \to 0$, $\delta_\nu \to 0$. Abbreviate $*_{\nu s} := *_{s_\nu + \delta_\nu s}$ and $X_{\nu s} := \delta_\nu X_{s_\nu + \delta_\nu s}$.

Lemma 3.1. *Let* $\Xi_\nu = A_\nu + \Phi_\nu \,\mathrm{d}s + \Psi_\nu \,\mathrm{d}t$ *be a sequence of solutions of the equation* (2), *with* $(*_s, X_s)$ *replaced by* $(*_{\nu s}, X_{\nu s})$, *on* $\Omega_\nu \times P$ *such that*

$$\sup_\nu \left(\varepsilon_\nu^{-1} \|F_{A_\nu}\|_{L^2(\Omega_\nu \times \Sigma)} + \|\partial_t A_\nu - \mathrm{d}_{A_\nu}\Psi_\nu\|_{L^2(\Omega_\nu \times \Sigma)}\right) < \infty, \tag{12}$$

$$\sup_\nu \left(\varepsilon_\nu^{-1} \|F_{A_\nu}\|_{L^\infty(\Omega_\nu \times \Sigma)} + \|\partial_t A_\nu - \mathrm{d}_{A_\nu}\Psi_\nu\|_{L^\infty(\Omega_\nu \times \Sigma)}\right) < \infty.$$

Then there are a subsequence, still denoted by Ξ_ν, a sequence of gauge transformations $g_\nu : \Omega_\nu \to \mathcal{G}(P)$, and a connection $\Xi_0 = A_0 + \Phi_0\, ds + \Psi_0\, dt$ on $\mathbb{C} \times P$ such that

$$\partial_t A_0 - d_{A_0}\Psi_0 + *_{s_0}(\partial_s A_0 - d_{A_0}\Phi_0) = 0, \qquad F_{A_0} = 0,$$
$$\lim_{\nu\to\infty}\left(\|g_\nu^* A_\nu - A_0\|_{L^\infty(K\times\Sigma)} + \sup_{(s,t)\in K} \left\|g_\nu^{-1} B_{\nu t} g_\nu - B_{0t}\right\|_{L^2(\Sigma)} \right) = 0$$

for every compact set $K \subset \mathbb{C}$; here $B_{\nu t} := \partial_t A_\nu - d_{A_\nu}\Psi_\nu$, $B_{0t} := \partial_t A_0 - d_{A_0}\Psi_0$.

Proof. For every compact set $K \subset \mathbb{C}$ there is a constant $\nu_K > 0$ such that, for every $(s,t) \in K$ and every $\nu \geq \nu_K$, there is a unique section $\eta_\nu(s,t) \in \Omega^0(\Sigma, \mathfrak{g}_P)$ such that

$$F_{A'_\nu} = 0, \qquad A'_\nu := A_\nu + *_{\nu s} d_{A_\nu}\eta_\nu,$$

and

$$\|d_{A_\nu}\eta_\nu\|_{L^\infty(\Sigma)} \leq c_1 \|F_{A_\nu}\|_{L^\infty(\Sigma)} \leq c_2 \varepsilon_\nu \tag{13}$$

(see Lemma 8.2 in [1]). Choose $\Phi'_\nu(s,t), \Psi'_\nu(s,t) \in \Omega^0(\Sigma, \mathfrak{g}_P)$ such that

$$d_{A'_\nu} *_{\nu s} \left(\partial_s A'_\nu - d_{A'_\nu}\Phi'_\nu - X_{\nu s}(A'_\nu)\right) = d_{A'_\nu} *_{\nu s} \left(\partial_t A'_\nu - d_{A'_\nu}\Psi'_\nu\right) = 0.$$

Note that the sequence $\Xi'_\nu = A'_\nu + \Phi'_\nu\, ds + \Psi'_\nu\, dt$ depends only on ν and not on the compact set K in question. One proves exactly as in [1, pp. 626, 627] that the sequence Ξ'_ν satisfies the estimates

$$\left\|\Xi'_\nu - \Xi_\nu\right\|_{1,p,\varepsilon;K} \leq c_{K,p}\varepsilon_\nu^{1+2/p}, \tag{14}$$
$$\left\|B'_{\nu t}\right\|_{L^\infty(K\times\Sigma)} \leq c_K, \tag{15}$$
$$\left\|B'_{\nu t} + *_{\nu s}\left(B'_{\nu s} - X_{\nu s}(A'_\nu)\right)\right\|_{L^p(K\times\Sigma)} \leq c_{K,p}\varepsilon_\nu^{1+2/p}, \tag{16}$$

for every compact set $K \subset \mathbb{C}$ and every $p \geq 2$, with suitable positive constants c_K and $c_{K,p}$. In addition we wish to prove the estimate

$$\sup_K \left\|B'_{\nu t} - B_{\nu t}\right\|_{L^2(\Sigma)} \leq c_K\sqrt{\varepsilon_\nu}. \tag{17}$$

To see this we use the identities

$$\begin{aligned} B'_t - B_t &= d_{A'}(\Psi' - \Psi) + *_s d_A \nabla_t\eta + *_s[B_t, \eta], \\ d_A *_s d_A(\Psi' - \Psi) &= d_A *_s B_t - [d_A B_t, \eta] - [F_A, \nabla_t\eta] \\ &\quad -[(A' - A) \wedge ([d_A\nabla_t\eta + [B_t, \eta])] \\ d_A *_s d_A\nabla_t\eta &= -d_A B_t - [d_A\nabla_t\eta \wedge d_A\eta] - [[B_t, \eta] \wedge d_A\eta] \\ &\quad -2[B_t \wedge *_s d_A\eta] - [d_A *_s B_t, \eta] \end{aligned} \tag{18}$$

(see (8.5), (8.7), and (8.8) in [1]). Here we have dropped the subscript ν. Since

$$d_A B_t = \nabla_t F_A, \qquad d_A *_s B_t = d_A B_s = \nabla_s F_A,$$

we obtain from Lemma 2.1 with $p = 2$ that, for every compact set $K \subset \mathbb{C}$, there is a constant $c'_K > 0$ such that

$$\sup_K \Big(\|\mathrm{d}_A B_t\|_{L^2(\Sigma)} + \|\mathrm{d}_A *_s B_t\|_{L^2(\Sigma)} \Big) \le c'_K \sqrt{\varepsilon}.$$

Hence it follows from (13) and the last equation in (18) that

$$\sup_K \|\mathrm{d}_A \nabla_t \eta\|_{L^2(\Sigma)} \le c''_K \sqrt{\varepsilon}.$$

Using this estimate and the second equation in (18) we obtain

$$\sup_K \big\|\mathrm{d}_A(\Psi' - \Psi)\big\|_{L^2(\Sigma)} \le c'''_K \sqrt{\varepsilon}.$$

Combining the last two estimates with the first equation in (18) we obtain (17). Now Ξ'_ν descends to a sequence

$$\bar{u}'_\nu : K \to \mathcal{M}(P)$$

of approximate holomorphic curves (see (16)) with uniformly bounded derivatives (see (15)). We must prove that the sequence $\bar{u}'_\nu$ is bounded in $W^{2,p}$ for some $p > 2$. By the elliptic bootstrapping analysis for holomorphic curves (see [3, App. B]), this is equivalent to a $W^{1,p}$-bound on $\bar{\partial}_J(\bar{u}'_\nu)$. To obtain such a bound we examine the following formula from [1, p. 627]:

$$\begin{aligned} B'_t + *_s(B'_s - X_s(A')) = {} & *_s \dot{*}_s \mathrm{d}_A \eta - [X_s(A), \eta] - *_s(X_s(A') - X_s(A)) \\ & + [(A' - A), \nabla_s \eta] - *_s[(A' - A), \nabla_t \eta] \\ & - \mathrm{d}_{A'}(\Psi' - \Psi + \nabla_s \eta) - *_s \mathrm{d}_{A'}(\Phi' - \Phi - \nabla_t \eta). \end{aligned} \tag{19}$$

To begin with, observe that, by Lemma 2.1, we have estimates of the form

$$\int_K \Big(\|\mathrm{d}_A B_t\|^p_{L^2(\Sigma)} + \|\mathrm{d}_A *_s B_t\|^p_{L^2(\Sigma)} \Big) \le c_{K,p} \varepsilon^p.$$

Carrying the argument in the proof of Lemma 2.1 one step further we obtain estimates for the second derivatives of the curvature and hence

$$\int_K \Big(\|\mathrm{d}_A \nabla_s B_t\|^p_{L^2(\Sigma)} + \|\mathrm{d}_A *_s \nabla_s B_t\|^p_{L^2(\Sigma)} \Big) \le c_{K,p};$$

similarly for ∇_t. Differentiate the identities in (18) to obtain

$$\int_K \Big(\|\mathrm{d}_A \nabla_s \nabla_s \eta\|^p_{L^2(\Sigma)} + \|\mathrm{d}_A \nabla_t \nabla_t \eta\|^p_{L^2(\Sigma)} + \|\mathrm{d}_A \nabla_s \nabla_t \eta\|^p_{L^2(\Sigma)} \Big) \le c_{K,p},$$

$$\int_K \Big(\big\|\mathrm{d}_A \nabla_s (\Psi' - \Psi)\big\|^p_{L^2(\Sigma)} + \big\|\mathrm{d}_A \nabla_t (\Psi' - \Psi)\big\|^p_{L^2(\Sigma)} \Big) \le c_{K,p}.$$

Combining these estimates with (19) we obtain

$$\int_K \big\|\nabla_s (B'_t + *_s(B'_s - X_s(A')))\big\|^p_{L^2(\Sigma)} \le c_{K,p},$$

and similarly for ∇_t. This is the required $W^{1,p}$-estimate for $\bar\partial_J(\bar u'_\nu)$. It follows that $\bar u'_\nu$ is bounded in $W^{2,p}$ and hence has a C^1-convergent subsequence. The limit of this subsequence is the required holomorphic curve in $\mathcal{M}(P)$. The assertion of the lemma now follows from (17) and the C^1-convergence of $\bar u'_\nu$. □

For Theorem 9.1. On page 630 replace the estimate in the assertion of Theorem 9.1 by (11) above. In the proof on page 631 replace the factor ε_ν^{-2} in (9.1) and (9.2) by ε_ν^{-1}. Replace the next displayed formula by

$$c_\nu = c_\nu(w_\nu) = \varepsilon_\nu^{-1}\left\|F_{A_\nu(w_\nu)}\right\|_{L^2(\Sigma)} + \left\|\partial_t A_\nu(w_\nu) - \mathrm{d}_{A_\nu(w_\nu)}\Psi_\nu(w_\nu)\right\|_{L^2(\Sigma)}.$$

On page 633 the assertion that the limits $A_\infty(\theta)$ and $\Phi_\infty(\theta)$ exist can be proved by a similar argument as in [2, Prop. 11.1]. Alternatively, one can use the beautiful and elegant argument in [4] for a direct proof of the energy identity.

On p. 634 replace the second displayed inequality by

$$\sup_{|w|\le\rho_\nu c_\nu}\left(\frac{1}{\varepsilon_\nu c_\nu}\left\|F_{\widetilde{A}_\nu(w)}\right\|_{L^2(\Sigma)} + \left\|\partial_t\widetilde{A}_\nu(w) - \mathrm{d}_{\widetilde{A}_\nu(w)}\widetilde{\Psi}_\nu(w)\right\|_{L^2(\Sigma)}\right) \le 2.$$

We prove that the limit connection Ξ_0 represents a nonconstant holomorphic sphere. First, note that

$$\frac{1}{\varepsilon_\nu c_\nu}\left\|F_{\widetilde{A}_\nu(0)}\right\|_{L^2(\Sigma)} + \left\|\partial_t\widetilde{A}_\nu(0) - \mathrm{d}_{\widetilde{A}_\nu(0)}\widetilde{\Psi}_\nu(0)\right\|_{L^2(\Sigma)} = 1$$

and use Corollary 1.1 with ε replaced by $\widetilde{\varepsilon}_\nu := \varepsilon_\nu c_\nu \to 0$ to deduce that the functions $\partial_t\widetilde{A}_\nu - \mathrm{d}_{\widetilde{A}_\nu}\widetilde{\Psi}_\nu$ and $(\varepsilon_\nu c_\nu)^{-1}F_{\widetilde{A}_\nu}$ are uniformly bounded on every compact subset of $\mathbb{C}\times\Sigma$. Second, use Lemma 3.1 to deduce that the sequence $\widetilde{\Xi}_\nu = \widetilde{A}_\nu + \widetilde{\Phi}_\nu \mathrm{d}s + \widetilde{\Psi}_\nu\,\mathrm{d}t$ has a C^1 convergent subsequence (after gauge transformation). Third, use Lemma 2.1 to deduce that $(\varepsilon_\nu c_\nu)^{-1}\|F_{\widetilde{A}_\nu(0)}\|_{L^2(\Sigma)} \to 0$ and hence

$$\left\|\partial_t A_0(0) - \mathrm{d}_{A_0(0)}\Psi_0(0)\right\|_{L^2(\Sigma)} = \lim_{\nu\to\infty}\left\|\partial_t\widetilde{A}_\nu(0) - \mathrm{d}_{\widetilde{A}_\nu(0)}\widetilde{\Psi}_\nu(0)\right\|_{L^2(\Sigma)} = 1.$$

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(Received October 4, 2004)