Annals of Mathematics **188** (2018), 1017–1018
https://doi.org/10.4007/annals.2018.188.3.6

Erratum for "Geometric Langlands duality and representations of algebraic groups over commutative rings"

By I. Mirković and K. Vilonen

Corrections for Section 12

In Section 12 of the article [MV07], we approximate $\widetilde{G}_\kappa$ by finite type quotients $\widetilde{G}^*_\kappa$ satisfying properties (12.11). One should not have imposed condition (12.11c) as at this point we do not yet know if any finite type quotients $\widetilde{G}^*_\kappa$ satisfying it exist. In the text it is erroneously claimed that (12.8) implies that $(\widetilde{G}_\kappa)_{\mathrm{red}}$ is of finite type when in reality (12.8) only asserts that $(\widetilde{G}_\kappa)_{\mathrm{red}}$ is of finite dimension. However, the argument in the paper goes through without imposing (12.11c) as follows. We write $\widetilde{G}_\kappa = \varprojlim \widetilde{G}^*_\kappa$ with the $\widetilde{G}^*_\kappa$ satisfying (12.11a) and (12.11b) and so that $\check{T}_\kappa \to \widetilde{G}_\kappa \to \widetilde{G}^*_\kappa$ is a closed embedding. Then we have $(\widetilde{G}_\kappa)_{\mathrm{red}} = \varprojlim (\widetilde{G}^*_\kappa)_{\mathrm{red}}$. At the end of Section 12 it is shown that all the $(\widetilde{G}^*_\kappa)_{\mathrm{red}}$ are connected, reductive and isomorphic to $\check{G}_\kappa$. If we have two finite type quotients $\widetilde{G}^*_\kappa$ and $\widetilde{G}^{**}_\kappa$ of $\widetilde{G}_\kappa$ satisfying (12.11a) and (12.11b) such that $\widetilde{G}_\kappa \to \widetilde{G}^{**}_\kappa \to \widetilde{G}^*_\kappa$, then the induced map $(\widetilde{G}^{**}_\kappa)_{\mathrm{red}} \to (\widetilde{G}^*_\kappa)_{\mathrm{red}}$ amounts to a surjective endomorphism of $\check{G}_\kappa$ that is an identity on the maximal torus $\check{T}_\kappa$. Such an endomorphism is a unipotent isogeny and, for example, by [PY06, 2.3 Corollary] is an isomorphism. Thus we conclude that $(\widetilde{G}_\kappa)_{\mathrm{red}} = \check{G}_\kappa$, which proves (12.6) and thus completes the identification of the group scheme $\widetilde{G}_{\mathbb{Z}}$ with $\check{G}_{\mathbb{Z}}$.

Remark. There are a few other places in the paper where things are not said quite correctly or where the arguments are too brief. These have been treated in the notes of Pierre Baumann and Simon Riche [BR18].

Keywords: Langlands duality, representation theory, algebraic groups
AMS Classification: Primary: 14L99.
KV thanks Geordie Williamson and Brian Conrad for helpful comments and suggestions and both authors thank the referees for very helpful comments.

References

[BR18] P. Baumann and S. Riche, Notes on the geometric Satake equivalence, in *Relative Aspects in Representation Theory, Langlands Functoriality and Automorphic Forms* (CIRM Jean-Morlet Chair, Spring 2016, LNM), *Lecture Notes in Math.* **2221**, Soc. Math. France, Paris, 2018, pp. 1–134. https://doi.org/10.1007/978-3-319-95231-4_1.

[MV07] I. Mirković and K. Vilonen, Geometric Langlands duality and representations of algebraic groups over commutative rings, *Ann. of Math.* (2) **166** no. 1 (2007), 95–143. MR 2342692. Zbl 1138.22013. https://doi.org/10.4007/annals.2007.166.95.

[PY06] G. Prasad and J.-K. Yu, On quasi-reductive group schemes, *J. Algebraic Geom.* **15** no. 3 (2006), 507–549, with an appendix by Brian Conrad. MR 2219847. Zbl 1112.14053. https://doi.org/10.1090/S1056-3911-06-00422-X.

(Received: November 18, 2017)
(Revised: July 4, 2018)

Department of Mathematics, University of Massachusetts,
Amherst, MA, USA
E-mail: mirkovic@math.umass.edu

School of Mathematics and Statistics, University of Melbourne,
VIC 3010, Australia and Department of Mathematics and Statistics,
University of Helsinki, Helsinki, Finland
E-mail: kari.vilonen@unimelb.edu.au,kari.vilonen@helsinki.fi