Annals of Mathematics **177** (2013), 783–785
http://dx.doi.org/10.4007/annals.2013.177.2.11

A correction to "Propagation of singularities for the wave equation on manifolds with corners"

By András Vasy

Abstract

We correct an error in the proof of Proposition 7.3 of the author's paper on the propagation of singularities for the wave equation on manifolds with corners. The correction does not affect the statement of Proposition 7.3, and it does not affect any other part of the paper.

There is a mistake in the proof of Proposition 7.3[1] of [1]; namely, a term was omitted in (7.9) so that the displayed equation after (7.15), as well as its analogues after (7.16), do not hold. The term omitted corresponds to the term $|x|^2$ in (7.8) being differentiated by the term $2A\xi \cdot \partial_x$ in the Hamilton vector field appearing in (6.3).

This mistake can be easily remedied as follows. First, after the displayed equation following (7.7) we specify one of the ρ_j slightly more carefully; namely, we require

$$\rho_1 = 1 - \tau^{-2}|\zeta|_y^2.$$

Note that $d\rho_1 \neq 0$ at q_0 for $\zeta \neq 0$ there. Then

$$|\tau^{-1} W^\flat \omega_0| \leq C_1' \omega_0^{1/2}(\omega_0^{1/2} + |t - t_0|)$$

still holds.

The argument of [1] proceeds with a motivational calculation, followed by the precise version of what is needed. We follow this approach here. So first the correct motivational calculation is presented.

We still have $p|_{x=0} = \tau^2 - |\xi|_y^2 - |\zeta|_y^2$. Thus, the equation after (7.9) can be strengthened to

$$\tau^{-2}|\xi|_y^2 \leq C(\tau^{-2}|p| + |x| + \omega_0^{1/2}),$$

This work is partially supported by NSF grant DMS-0801226.

[1]All equation and proposition numbers of the form (7.xx) or 7.xx refer to [1].

i.e., with $|t-t_0|$ dropped, using that $|\rho_1| = |1-\tau^{-2}|\zeta|^2_y| \leq \omega_0^{1/2}$. The analogue of (7.9) for ω_0 in place of ω still holds:

$$\begin{aligned}|\tau^{-1}H_p\omega_0| &\leq \tilde{C}_1''\omega_0^{1/2}(\omega_0^{1/2}+|x|+|t-t_0|+\tau^{-2}|\xi|^2)\\ &\leq C_1''\omega^{1/2}(\omega^{1/2}+|t-t_0|+\tau^{-2}|p|).\end{aligned}$$

But we also have (and this was the dropped expression)

$$|\tau^{-1}H_p|x|^2| \leq \tilde{C}_1'|x|\,(|x|+|\tau|^{-1}\,|\xi|) \leq C_1'\omega^{1/2}(\omega^{1/2}+(\tau^{-2}|p|+\omega^{1/2})^{1/2}).$$

Thus, the displayed equation after (7.15) becomes (at $p=0$), with $C_1 = C_1'+C_1''$,

$$\begin{aligned}\tau^{-1}H_p\phi &= H_p(t-t_0)+\frac{1}{\epsilon^2\delta}H_p\omega\\ &\geq c_0/2-\frac{1}{\epsilon^2\delta}C_1\omega^{1/2}(\omega^{1/2}+|t-t_0|+\omega^{1/4})\\ &\geq c_0/2-4C_1\left(\delta+\frac{\delta}{\epsilon}+\left(\frac{\delta}{\epsilon}\right)^{1/2}\right)\geq c_0/4>0\end{aligned}$$

provided that $\delta < \frac{c_0}{64C_1}$, $\frac{\epsilon}{\delta} > \max(\frac{64C_1}{c_0}, (\frac{64C_1}{c_0})^2)$, i.e., that δ is small, but ϵ/δ is not too small — roughly, ϵ can go to 0 at most proportionally to δ (with an appropriate constant) as $\delta\to 0$. The rest of the rough argument then goes through.

The precise version is similar. In (7.10) the estimate on the f_i term must be weakened:

$$\begin{gathered}\tau^{-1}H_p\omega = f_0+\sum_i f_i\tau^{-1}\xi_i+\sum_{i,j}f_{ij}\tau^{-2}\xi_i\xi_j,\\ f_i, f_{ij}\in\mathcal{C}^\infty({}^{\mathrm{b}}T^*X),\quad |f_i|\leq C_1\omega^{1/2},\quad |f_{ij}|\leq C_1\omega^{1/2},\end{gathered}$$

f_i, f_{ij} homogeneous of degree 0. This affects the estimates on r_i below (7.16):

$$|r_0|\leq\frac{C_2}{\epsilon^2\delta}\omega^{1/2}(|t-t_0|+\omega^{1/2}),\quad |\tau r_i|\leq\frac{C_2}{\epsilon^2\delta}\omega^{1/2},\quad |\tau^2 r_{ij}|\leq\frac{C_2}{\epsilon^2\delta}\omega^{1/2},$$

and $\operatorname{supp} r_i$ lying in $\omega^{1/2}\leq 3\epsilon\delta$, $|t-t_0|<3\delta$. Thus,

$$|r_0|\leq 3C_2\left(\delta+\frac{\delta}{\epsilon}\right),\quad |\tau r_i|\leq 3C_2\epsilon^{-1},\quad |\tau^2 r_{ij}|\leq 3C_2\epsilon^{-1}.$$

Thus, only the R_i term needs to be treated differently from [1]. We again let $T\in\Psi_{\mathrm{b}}^{-1}(X)$ be elliptic with principal symbol $|\tau|^{-1}$ near $\dot{\Sigma}$ (more precisely, on a neighborhood of $\operatorname{supp} a$), $T^-\in\Psi_{\mathrm{b}}^1(X)$ a parametrix, so $T^-T=\mathrm{Id}+F$, $F\in\Psi_{\mathrm{b}}^{-\infty}(X)$. Then there exists $R_i'\in\Psi_{\mathrm{b}}^{-1}(X)$ such that for any $\gamma>0$,

$$\begin{aligned}\|R_iw\| = \|R_i(T^-T-F)w\| &\leq \|(R_iT^-)(Tw)\|+\|R_iFw\|\\ &\leq 6C_2\epsilon^{-1}\|Tw\|+\|R_i'Tw\|+\|R_iFw\|\end{aligned}$$

for all w with $Tw \in L^2(X)$; hence,

$$\begin{aligned}|\langle R_i D_{x_i} v, v\rangle| \leq & 6C_2\epsilon^{-1}\|TD_{x_i}v\|\,\|v\| \\ & + 2\gamma\|v\|^2 + \gamma^{-1}\|R_i' T D_{x_i} v\|^2 + \gamma^{-1}\|F_i D_{x_i} v\|^2,\end{aligned}$$

with $F_i \in \Psi_{\mathrm{b}}^{-\infty}(X)$. Now we use that R_i is microlocalized in an $\epsilon\delta$-neighborhood of $\mathcal{G}$, rather than merely a δ-neighborhood, as in [1], due to the more careful choice of ρ_1: $\mathcal{G}$ is given by $\rho_1 = 0$, $x = 0$, and we are microlocalized to the region where $|\rho_1| \leq 3\epsilon\delta$, $|x| \leq 3\epsilon\delta$. For $v = \tilde{B}_r u$, $\tilde{B}_r = \tilde{B}\Lambda_r$, Lemma 7.1 thus gives (taking into account that we need to estimate $\|TD_{x_i}v\|$ rather than its square)

$$\begin{aligned}|\langle R_i D_{x_i} v, v\rangle| \leq & \; 6C_2'\epsilon^{-1}(\epsilon\delta)^{1/2}\|\tilde{B}_r u\|^2 \\ & + C_0\gamma^{-1}\Big(\|G\tilde{B}_r u\|^2_{H^1(X)} + \|\tilde{B}_r u\|^2_{H^1_{\mathrm{loc}}(X)} + \|\tilde{G}Pu\|^2_{H^{-1}(X)} + \|Pu\|^2_{H^1_{\mathrm{loc}}(X)}\Big) \\ & + 3\gamma\|\tilde{B}_r u\|^2 + \gamma^{-1}\|R_i' T D_{x_i}\tilde{B}_r u\|^2 + \gamma^{-1}\|F_i D_{x_i}\tilde{B}_r u\|^2,\end{aligned}$$

where the first term is the main change compared to [1]. Its coefficient, $(\delta/\epsilon)^{1/2}$, means that it can then be handled exactly as the R_{ij} term in [1], thus completing the proof.

References

[1] A. Vasy, Propagation of singularities for the wave equation on manifolds with corners, *Ann. of Math.* **168** (2008), 749–812. MR 2456883. Zbl 1171.58007. http://dx.doi.org/10.4007/annals.2008.168.749.

(Received: December 1, 2008)

Stanford University, Stanford, CA
E-mail: andras@math.stanford.edu