Corrigendum to “A smooth foliation of the 5-sphere by complex surfaces”

By Laurent Meersseman and Alberto Verjovsky

Our paper [MV02] claims that it describes a foliation of $S^5$ by complex surfaces. However it was pointed out to us by the anonymous referee of a related article that the foliation constructed in the paper lives in fact on a 5-manifold with nontrivial fundamental group.

We observe that, even with this modification, this foliation is still the first example of such an exotic CR-structure.

We use the notations and results of [MV02] and assume that the reader is acquainted with them. We refer to [MV11] for full details of what follows.

The foliation of [MV02] is obtained by gluing, thanks to Lemma 1, two tame foliations on manifolds with boundary. The second one, $\mathcal{N}$, is supposed to be diffeomorphic to $K \times \mathbb{D}$. However, the foliated $\mathcal{N}$ which is constructed by quotient is in fact diffeomorphic to the $\mathbb{D}$-bundle associated to $L$. They are definitely not diffeomorphic, since the first manifold has a nilpotent fundamental group (see [MV02, §1.2]), whereas the second one retracts on $S^1 \times S^1 \times S^1$.

When gluing $\mathcal{M}$ and this “new” $\mathcal{N}$, one does not obtain the 5-sphere. Let

$$F^p = \{(z_0 : z_1 : z_2 : z_3) \in \mathbb{P}^3 \mid z_1^3 + z_2^3 + z_3^3 = z_0^3\}.$$ 

Then, what is really proved in [MV02] is the following theorem.

**Theorem.** Let $Z$ be the 5-dimensional bundle over the circle with fiber $F^p$ and monodromy the multiplication by $\omega = \exp 2i\pi/3$ on the affine part.

There exists on $Z$ an exotic smooth, codimension-one, integrable and Levi-flat CR-structure. The induced foliation by complex surfaces satisfies:

(i) There are only two compact leaves both biholomorphic to an elliptic bundle over $E_\omega$. Since this surface has odd first Betti number it is not Kähler.

(ii) One compact leaf is the boundary of a compact set in $Z$ whose interior is foliated by line bundles $L$ over $E_\omega$ with Chern class $-3$. The two compact leaves are the boundary components of a collar and the leaves in the interior of this collar are biholomorphic to $W$, the principal $\mathbb{C}^*$-bundle associated to $L$. 

1951
(iii) The other leaves have the homotopy type of a bouquet of eight copies of $S^2$ and they are all biholomorphic to the affine complex smooth manifold $F^p \cap \mathbb{C}^3$.

References


(Received: May 6, 2011)
(Revised: June 13, 2011)

I.M.B., Université de Bourgogne, Dijon Cedex, France
E-mail: laurent.meersseman@u-bourgogne.fr

Instituto de Matemáticas de la UNAM, Unidad Cuernavaca, Cuernavaca, Morelos, México
E-mail: alberto@matcuer.unam.mx